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A unified equation for predicting traction for wheels on sand over a range of braked, towed, and powered operations

George L. Mason [a], James M. Williams [a,b], Farshid Vahedifard [a,b,*], Jody D. Priddy [c]

[a] Center for Advanced Vehicular Systems (CAVS), Mississippi State University, Mississippi State, MS 39762, USA
[b] Dept. of Civil and Environmental Engineering, Mississippi State University, Mississippi State, MS 39762, USA
[c] U.S. Army Engineer Research and Development Center (ERDC), 3909 Halls Ferry Road, Vicksburg, MS 39180. USA

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A B S T R A C T

Vehicle traction between the wheel and the ground surface is a critical design element for on-road and off-road mobility. Adequate traction in dry sand relates to the vehicle's ability to negotiate deserts, sand dunes, climb slopes, and ingress/egress along beaches. The existing traction equations predict values for only one mode of operation (braked, towed, or powered). In this article, we propose a unified algorithm for continuous prediction of traction over a range of braked, towed, and powered operations for wheels operating on sand. A database of laboratory and field records for wheeled vehicles, entitled Database Records for Off-road Vehicle Environments (DROVE), was used to develop the proposed algorithm. The algorithm employs the ratio of contact pressure to cone index as a primary variable to develop fitting parameters for a relationship between slip and traction. The performance of the algorithm is examined versus the measured data and is also compared against two alternative equations. The new equation showed higher correlation and lower error compared to the existing equations for powered wheels. The proposed equation can be readily implemented into off-road mobility models, eliminating the need for multiple traction equations for different modes of operation.

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1. Introduction

Off-road vehicle design and analysis, as well as optimal route planning, require the prediction of key traction parameters. The net traction, or drawbar pull (DBP), is often optimized for design or path selection purposes. The DBP is the pulling force produced by the gross traction (T) after taking the motion resistance (MR) forces acting on the wheel into account. The tractive performance is a function of the size of the contact patch of the tire on the soil surface, the torque acting on the tire, and the slip (i) experienced during operation. This contact area of the tire on the ground is a function of the tire's effective radius and sinkage (z) (Mason et al., 2016). Traction tests in the past have been conducted to evaluate the effects of changes in velocity (Coutermarsh, 2007), steering (Durham, 1976), and variations in tire characteristics (Turnage, 1995). Likewise, MR is affected by velocity, slip, turning, loading, tire inflation pressure, and sinkage (Taghavifar and Mardani, 2013; Coutermarsh, 2007; Brixius, 1987; Sharma and Pandey, 2001). Due to the number of considerations required, prediction of tractive forces remains a key area of study.

A variety of methods, including empirical relationships, semi-empirical relationships, or physics-based discrete element or finite element numerical models, use soil-tire inputs for prediction of tractive forces (Tiwari et al., 2010; Taheri et al., 2015; Du et al., 2017). While the physics-based models provide high fidelity results and allow consideration of the many forces acting on the tire, their computational cost and need for a significant number of input parameters make such methods difficult to implement. Therefore, empirical and semi-empirical relationships have often been used to correlate more easily obtained measurements to vehicle performance. Many proposed empirical algorithms rely on a dimensionless wheel mobility number (Hegazy and Sandu, 2013). The mobility number is typically comprised of tire and soil characteristics, which drive the tire traction models that have been developed (e.g., Brixius, 1987; Elwaleed et al., 2006; Maclaurin, 2014). Several of these empirical and semi-empirical relationships have been implemented in the Vehicle Terrain Interface (VTI) model, a high-resolution empirical model that addresses the interactions at the traction-terrain element interface. The VTI serves as a practical tool to support virtual prototyping of off-road vehicles

* Corresponding author at: Dept. of Civil and Environmental Engineering, Mississippi State University, Mississippi State, MS 39762, USA.

E-mail addresses: mason@cavs.msstate.edu (G.L. Mason), jmw835@msstate.edu (J.M. Williams), farshid@cee.msstate.edu (F. Vahedifard), Jody.D.Priddy@usace.army.mil (J.D. Priddy).

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Nomenclature

δ	tire deflection [m]	G	cone index gradient [kN/m^2]
A	fitting parameter related to intercept [–]	h	tire section height [m]
B	fitting parameter related to slope [–]	i	slip [%]
b	tire section width [m]	MR	motion resistance [–]
B_n	Brixius number [–]	N_s	sand numeric [–]
C	fitting parameter related to inflection point [–]	Q	torque [N m]
CA	area of tire contact patch [m^2]	T	gross traction [–]
CI	cone index [kPa]	V_a	actual wheel velocity [m/s]
CP	contact pressure [kPa]	V_t	theoretical wheel velocity [m/s]
d	tire overall diameter [m]	W	wheel load [kN]
DBP	drawbar pull [–]	z	sinkage [m]

without the computational cost of the larger models, a valuable part of the computer-aided design process. Evaluation of VTI algorithms using the Database Records for Off-road Vehicle Environments (DROVE) indicated a need for further improvement of traction equations (Vahedifard et al., 2016, 2017).

The majority of existing traction equations predicts values for only one mode of operation. This work presents a new, unified algorithm that can continuously predict traction over a range of braked, towed, and powered operations for wheels operating on sand. This feature eliminates the need for using multiple equations found in other empirical models such as the VTI. The new algorithm is developed using laboratory and field records available in the DROVE dataset. Previous studies using DROVE have already proposed new calibrations for existing VTI equations (Dettwiller et al., 2017, 2018), as well as new equations for z (Mason et al., 2016) and MR (Williams et al., 2017), but this work is the first to apply DROVE to the development of a new traction equation. The proposed equation takes a continuous s-shaped form for the traction-slip relationship. The y-intercept, slope, inflection point, minimum, and maximum values are required to establish the gross traction-slip relationship. Such information can be determined using a variety of methods including empirical relationships based on test data or the application of discrete element models. For the purpose of this work, the filtered DROVE dataset was used to develop relationships between these fitting parameters and the ratio of contact pressure (CP) to cone index (CI). Information on the DROVE dataset used is provided in the following section. The form of the equation and its development are then presented. For comparison and evaluation of the proposed equation, existing traction equations are also presented and their performance is compared to the new equation.

2. Database Records for Off-road Vehicle Environments 1.0 (DROVE)

A substantial number of test results are required to test and develop prediction equations. A recently created database, DROVE (Vahedifard et al., 2016, 2017), containing historic test data was selected to provide the necessary information. The first version of DROVE (DROVE 1.0) includes records for field and laboratory tests of wheeled vehicles conducted on dry sands (Vahedifard et al., 2016) and wet clays (Vahedifard et al., 2017). Each test record includes all relevant information known about an individual test. DROVE records commonly contain tire parameters such as tire width (b), diameter (d), section height (h), deflection (δ), and load (W) corresponding to a given set of performance records over a given soil strength. Typically, recorded performance parameters included at least one of the key traction parameters (DBP or MR), z, i, and torque (Q). Soil strength for DROVE records is in terms of CI for the top 150 mm of soil. Coarse-grained predictive equations often use the gradient of cone index (G). For the top 150 mm, dividing CI by 3.47 provides an approximate G value (Turnage, 1995). The test records in DROVE 1.0 were conducted over Yuma sand, river wash sand, and mortar sand over the course of several decades. The gradation of these sands is shown in Fig. 1. Each of the sands was dry (less than 1% moisture content by weight) throughout all test records.

For this study, the sand dataset from DROVE 1.0 (Vahedifard et al., 2016) was filtered to remove records that did not include torque (Q). This was done to ensure that each record used in the development of the new equation could have an approximate measured gross traction coefficient value based on the radius, Q, and W (Priddy, 1999):

$$\frac{T}{W} = \frac{Q}{(d/2)W} \tag{1}$$

This approximation was used in place of measured gross traction as the DROVE database, and does not contain direct records of gross traction. These filtered data were then used to develop and test a new equation. In order to evaluate the developed equation's performance, existing traction equations were applied to the same dataset and the root mean square error (RMSE) of each of the predictions against the measured values was determined.

3. Selected functional form

The theoretical correlation between slip and the traction has been proposed by a number of authors (e.g., Wismer and Luth, 1973; Karafiath and Nowatzki, 1978) and is illustrated in Fig. 2. For the purposes of this work, the relationship considers traction as a coefficient produced by dividing the gross traction by the applied load. The traction-slip relationship can be most easily described as an "s-shaped" curve. In addition to the conceptual relationship, illustrative figures of the braked and powered operation are also presented in Fig. 2. The transition zone in Fig. 2 is the region in which the tire may be operating in the powered, towed, or braked mode, and additional information regarding its operation would be necessary to determine which mode is active. The conceptual trends depicted in Fig. 2 demonstrate a curved function with horizontal asymptotes at the minimum and maximum traction values. The exact placement of these values is dependent upon the contact pressure of the wheel and the soil strength.

Existing traction prediction methods are often valid for only a single mode of operation (i.e., braked, towed, or powered). However, the ability to predict the tractive forces in all three modes of operation is necessary. Therefore, a characteristic curve equation, based on the continuous theoretical relationship illustrated in Fig. 2 would be ideal. Due to the theoretical shape of this

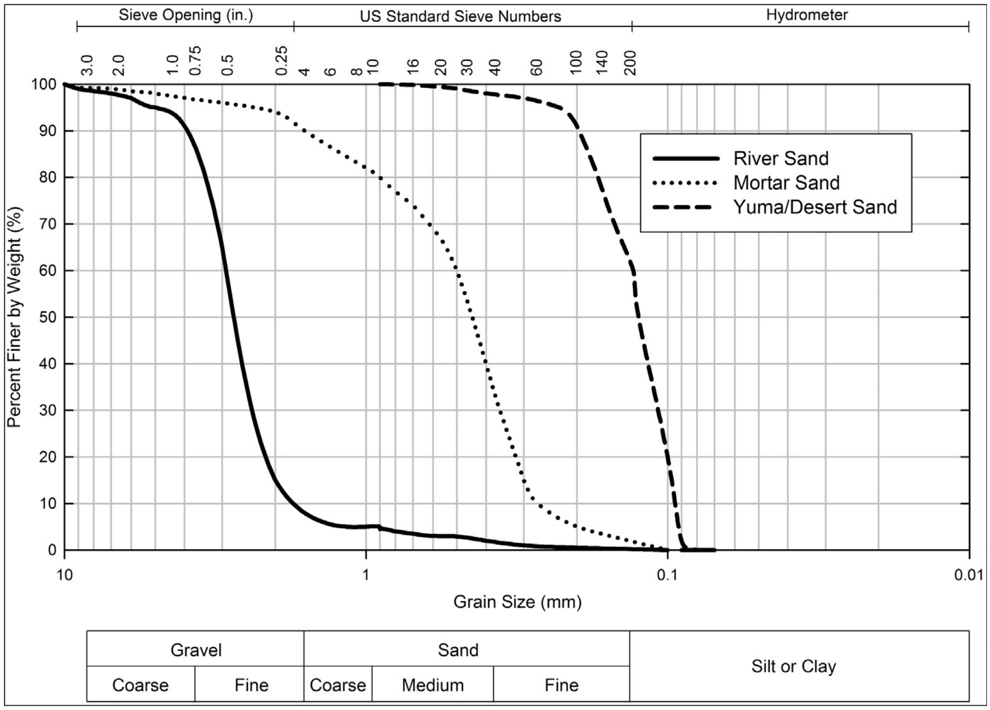

Fig. 1. Typical grain size distributions of the sands included in DROVE. 1 in. = 25.4 mm.

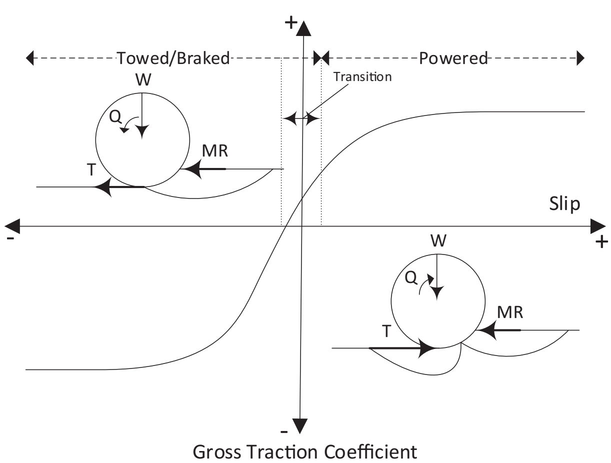

Fig. 2. Conceptual relationship between gross traction coefficient and slip. The transition zone is the region in which the tire may be operating in the powered, towed, or braked mode, and additional information regarding its operation would be necessary to determine which mode is active.

relationship it can be estimated using a general logistic function. The general form of the logistic function is described by Eq. (2):

$$Y(x) = Minimum + \frac{Maximum - Minimum}{(1 + Ae^{-Bx})^{1/C}} \tag{2}$$

where: Y(x) is the dependent variable, Minimum is the minimum bound, Maximum is the maximum bound, A = is a fitting parameter related to the intercept of the equation, B is the slope or growth rate parameter, x is the independent variable, and C is a fitting parameter which affects the location of the maximum growth. This form was selected due to the use of maximum and minimum bounds similar to those that bound the conceptual relationship. Furthermore, the A parameter allows for adjustment of the y-intercept which is comparable to the traction coefficient at zero slip.

To show that the data follow the basic trend illustrated in Fig. 2 and can be captured by a generalized logistic function as presented in Eq. (2), a plot of the traction coefficient versus slip was generated from the data available in DROVE. The definition of the i used (see Eqs. (3) and (4) was dependent upon the type of operation. Negative slip values, referred to as skid slip, were computed using Eq. (3) instead of the slip definition provided (ISTVS Standards, 1977) in Eq. (4). Both of these equations use the theoretical wheel velocity (V_t) based on the wheel angular velocity and wheel rolling radius, and the actual forward velocity of the vehicle (V_a) to define slip:

$$i = \frac{V_a/V_t}{V_a} \qquad Skid \tag{3}$$

$$i = \frac{V_t/V_a}{V_t} \qquad Powered \tag{4}$$

For this plot, the data were broken into bins of contact pressure to cone index ratios in order to evaluate their potential in defining relationships for the fitting parameters required for Eq. (2). The completed plot is presented in Fig. 3, and demonstrates the expected trend and the influence of the CP to CI ratio in the powered range of operation.

4. Development of the unified gross traction equation

Based on Figs. 2 and 3 the gross traction coefficient can be represented by rewriting Eq. (2) replacing x with slip and Y(x) with the gross traction coefficient resulting in Eq. (5):

$$\frac{T}{W} = Minimum + \frac{Maximum - Minimum}{(1 + Ae^{-Bi})^{1/C}} \tag{5}$$

The Minimum and Maximum in Eq. (2) refer to the maximum and minimum gross traction coefficients that will bound the unified equation, while A, B, and C provide the same fitting effects as presented in Eq. (5). For the purpose of this study, the minimum,

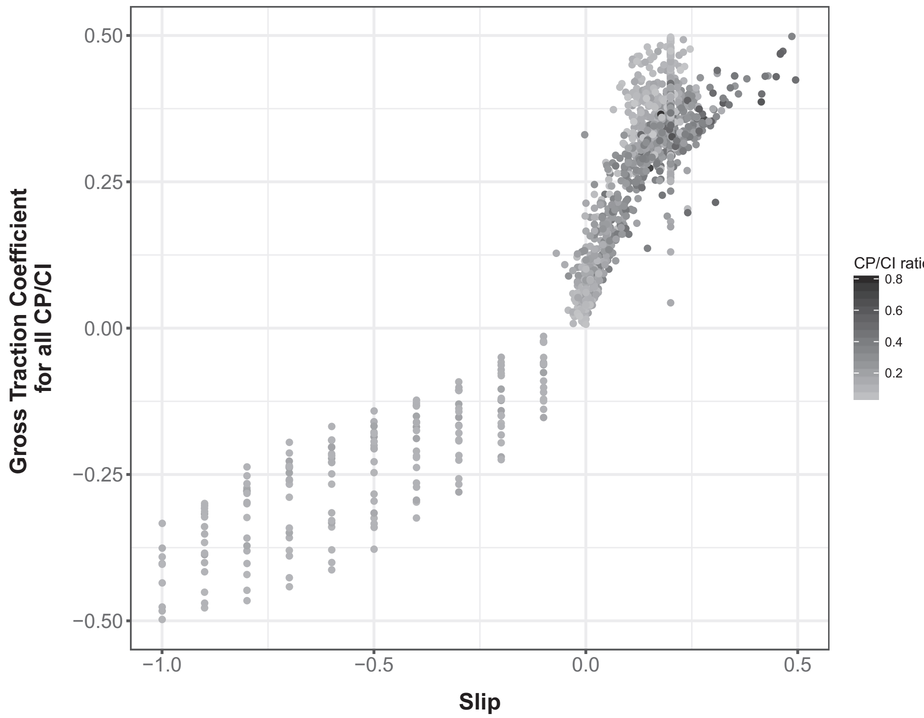

Fig. 3. Measured gross traction coefficient and slip for various CP to CI ratios.

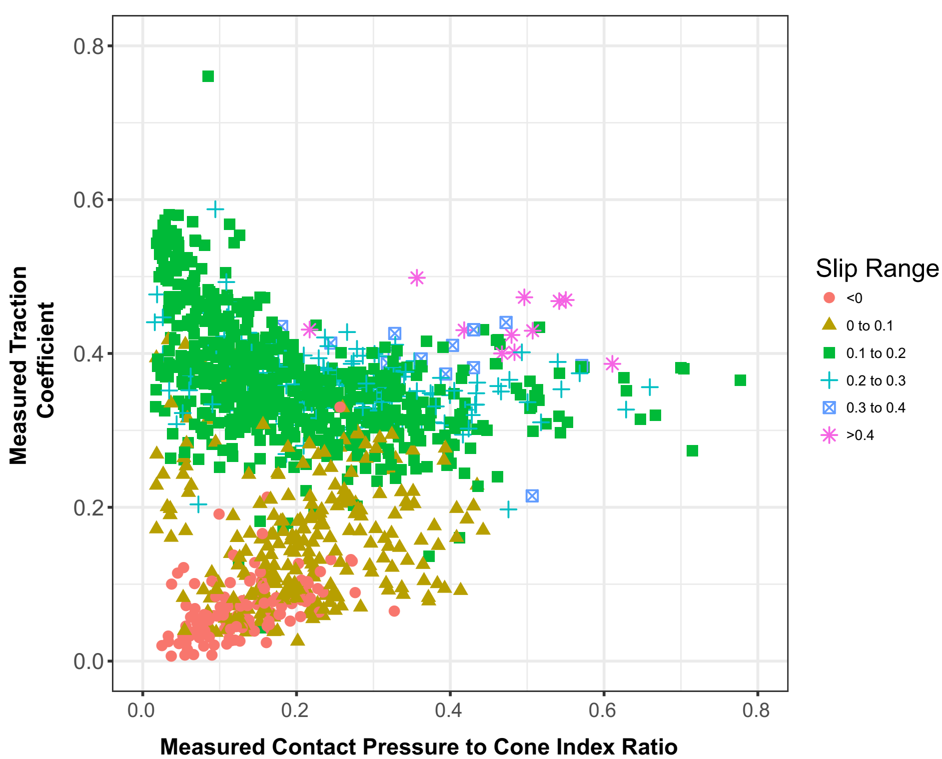

Fig. 4. Measured gross traction coefficient versus CP to CI ratio for powered records.

maximum, and fitting parameters can be expressed by a relationship to the CP to CI ratio. The CP is the ratio of the vertical load to the contact area (CA) of the tire as measured on a rigid flat surface (ISTVS Standards, 1977). Empirical relationships were developed based on trends in the DROVE dataset as described in the following subsections.

4.1. Minimum gross traction relationship

The minimum tractive force for a wheel occurs during negative slip or braking. Negative wheel slip is when the theoretical wheel velocity is less than the forward velocity of the vehicle. Such conditions may occur for unpowered wheels, unbalanced loads

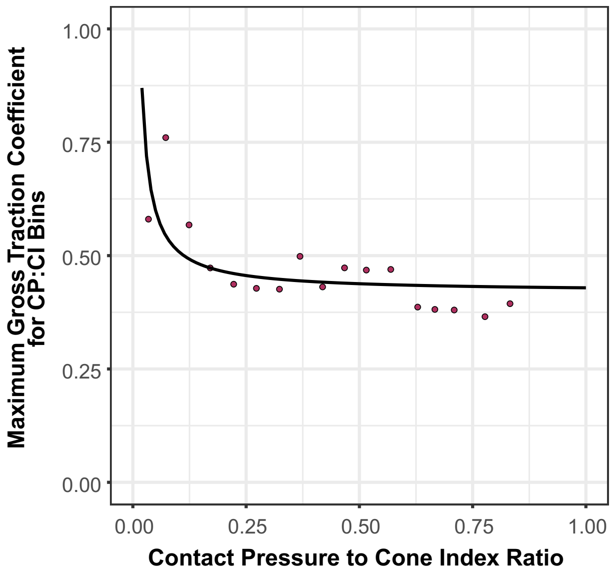

Fig. 5. Fitting of maximum gross traction coefficient relationship based on CP to CI ratio.

(unequal vertical loads applied to different wheels of the same vehicle), or with certain traction control systems. In addition to these conditions, braked wheels also have larger forward velocities than the theoretical wheel speed based on the angular velocity. As previously mentioned, the induced slip is referred to a skid under these braked conditions. In an effort to support design lengths of unpaved runways, Kraft et al. (1971a,b) conducted a series of braking tests. The Kraft data set included in DROVE provides a source for defining negative traction at select contact pressures and soil strengths. However, this dataset has a very limited range of CP to CI ratios, thereby preventing any significant trends in the minimum values from being determined. Based on the available data, the minimum traction coefficient is approximately −0.5 and occurs when braking slip is more negative than −60%. Additional testing with higher CP to CI ratios could provide more insight to this fitting parameter, but for the purposes of this study a constant value of −0.5 was selected.

4.2. Maximum gross traction relationship

The maximum gross traction coefficient occurs during powered operation. To evaluate potential relationships a plot of the measured gross traction coefficients versus the CP to CI ratio binned by slip was produced as presented in Fig. 4. Based on Fig. 4, the maximum traction occurs at slips greater than 10% for the entire range of CP to CI ratios. Furthermore, the maximum traction decreases as the CP to CI ratio increases from 0 to 0.2, and then approaches a horizontal asymptote beyond 0.2. It is possible that for CP to CI ratios greater than those contained in DROVE the relationship may change further, but for this work a reciprocal function was fit to the available data. The reciprocal function was selected in order to capture the horizontal asymptote present as the traction coefficient approached 0.45.

In order to fit this function, bins of CP to CI ratios with a range of 0.05 were produced. From these bins the maximum gross traction coefficient and median CP to CI ratio for each bin were determined. The values for each bin were then used to fit a reciprocal function to define the maximum traction coefficient based on the CP to CI ratio resulting in Eq. (6):

$$Maximum = 0.42 + 0.009\left(\frac{1}{CP/CI}\right) \tag{6}$$

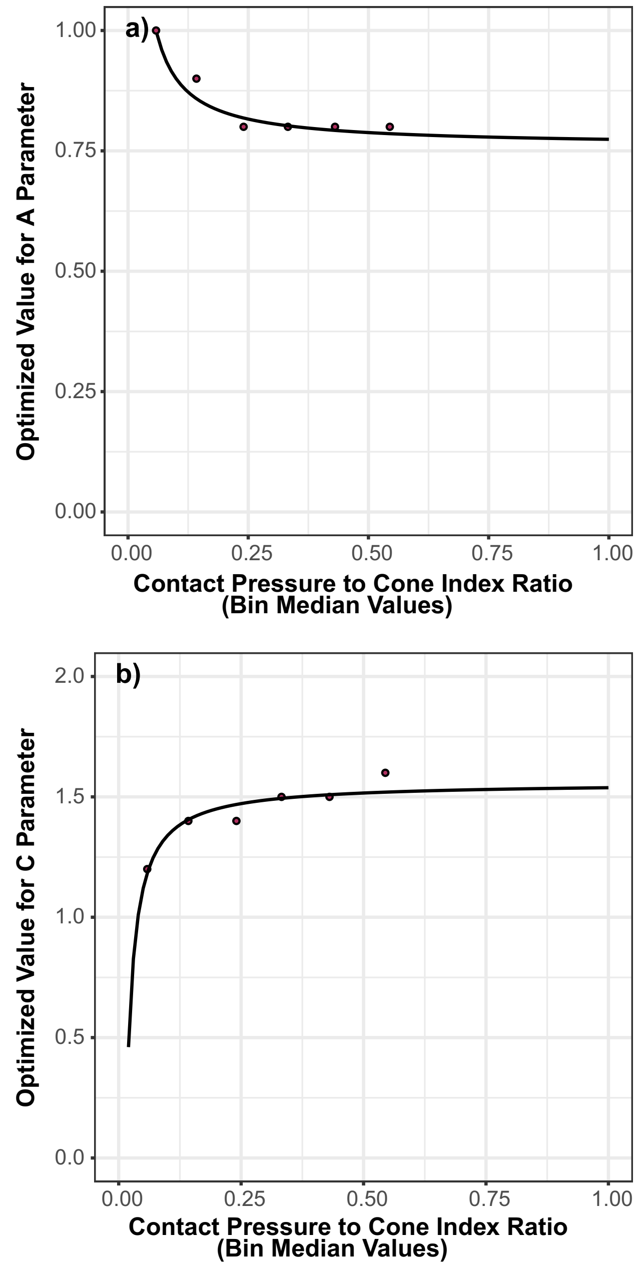

Fig. 6. Empirical fitting of parameters (a) A and (b) C based on the CP to CI ratio.

The resulting equation had an R^2 of 0.413 and a p-value of 0.003, indicating the regression was significant. The resulting equation is plotted, with the data used in the fitting process, in Fig. 5.

4.3. Unified equation fitting Parameters.

After defining the minimum and maximum gross traction coefficients based on the CP to CI ratio, the A, B, and C fitting parameters were also fit based on an optimization of the RMSE. For this process, the DROVE dataset was once again divided into bins of CP to CI ratios, this time using bin widths of 0.1. The effect of changing each fitting parameter on the RMSE was evaluated in each bin. The fitting parameter B, related to the slope of the equation, was found to have a limited effect when the value was placed between 6 and 11, and did not present any relationship to the bin being optimized. Therefore, slope was set to a constant value of 7

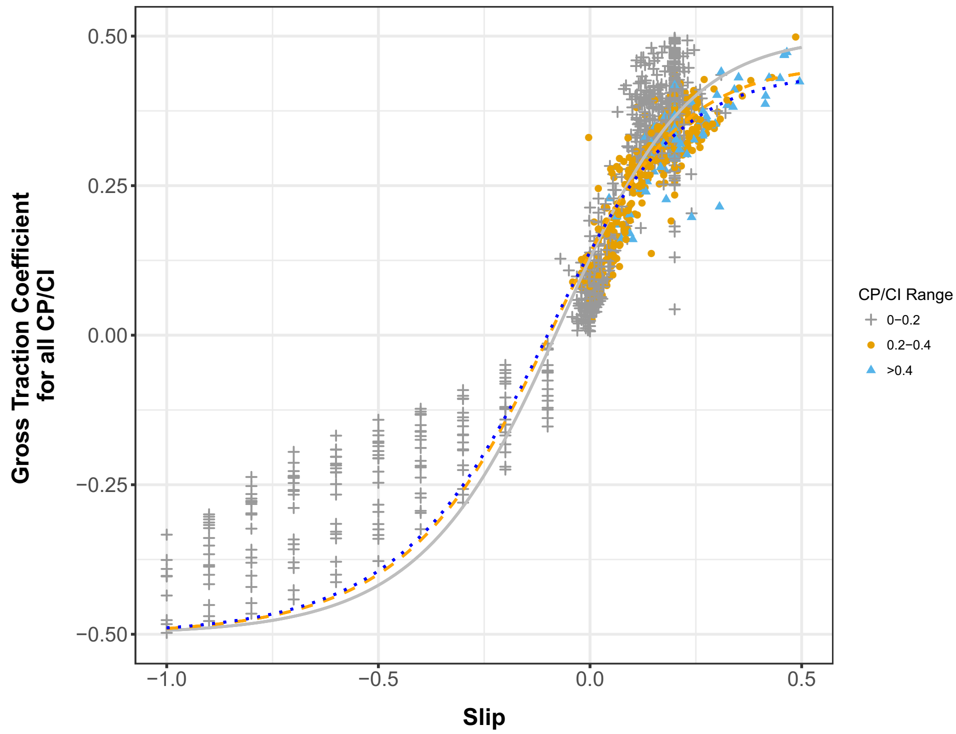

Fig. 7. Gross traction coefficient as a function of slip for ranges of CP to CI ratios.

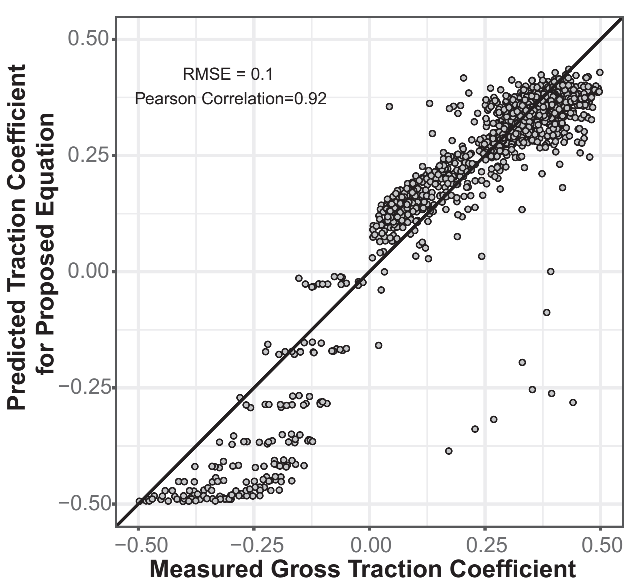

Fig. 8. Predicted versus measured gross traction coefficient for all modes of operation. Straight line in the graph is the 1:1 (y = x) line.

and the other two parameters were once again optimized. Following optimization, the determined A and C values were taken for each bin along with the median CP to CI ratio for each bin. Similar to the maximum gross traction parameter, these values were used to develop equations defining the values as a function of the CP to CI ratio. Based on the plot of the data, a reciprocal function was once again used for the fitting process. The selected values resulted in Eqs. (7) and (8):

$$A = 0.76 + 0.014\left(\frac{1}{CP/CI}\right) \tag{7}$$

$$C = 1.56 - 0.022\left(\frac{1}{CP/CI}\right) \tag{8}$$

The resulting equations for A and C provided p-values of 0.001 and 0.005, respectively; and R^2 values of 0.93 and 0.85, respectively. The p-values indicate that the trends were significant. The resulting equations are plotted, with the data used in the fitting process, in Fig. 6a and b.

5. Performance of unified gross traction equation

To illustrate the performance of this functional form and the determined empirical fitting parameters, Fig. 3 was recreated using three bins of CP to CI ratios: 0–0.2, 0.2–0.4, and greater than 0.4. In addition to the binned gross traction coefficient and slip data, the developed equation was applied to the median CP to CI ratio for each bin and added to the plot as a smooth curve. The final version of this plot is presented in Fig. 7 and contains 1500 unique records.

Overall, Fig. 7 illustrates that the developed equation follows both the theoretical form presented in Fig. 2 as well as the actual trends in the data. It is noted that the performance of the developed equation does not match the braked data trends accurately and typically predicts a lower gross traction coefficient. For braked operations this can be considered an overestimation of the braking traction. This trend to over-predict braking gross traction coefficient is further illustrated in Fig. 8 which provides the predicted versus measured gross traction coefficient for all operation types. Fig. 8 contains a 1:1 line, which is a y = x line, where the predicted and measured values match. The over-prediction is likely due to the limited number of braked data in DROVE. Another potential cause for this poor performance is the change in the definition of slip in this range and the addition of a correction factor may assist in this region.

Within the powered region the data point and curves also indicate that the CP to CI ratio alone does not fully separate the data. The use of additional tire information may have the potential to provide further insight to the powered trends. The use of CP to CI ratio to create empirical relationships for the fitting parameters further limits the effectiveness of this equation as the braked data have a limited range of values and no powered information exceeded a CP to CI ratio of 0.9. A final limitation of this relationship is related to the powered range of i. The DROVE dataset has values of i greater than 0.5, but this information did not provide the torque values necessary to generate gross traction. Therefore, the presented equation has only been presented for powered i ranging from −100 to 50%. Despite these limitations, the authors believe the proposed form provides utility in the form of a framework for the relationship between gross traction and slip that can be further improved. Further research is recommended to examine the fitting of equations outside of the current range.

6. Existing gross traction coefficient equations

In order to further evaluate the proposed equation, existing equations were applied to the DROVE database for comparison. Existing equations considered in this study include the gross traction equation from the VTI and the equation proposed by Brixius (1987).

6.1. VTI traction equation

The VTI equations are based on a dimensionless soil numeric computed using W, h, b, d, δ, and G as follows (Jones et al., 2015):

$$N_s = \frac{G(bd)^{\frac{3}{2}}\delta}{Wh} \tag{9}$$

where N_s is the sand numeric and can be used along with slip to predict the gross traction coefficient (Jones et al., 2015):

$$\frac{T}{W} = X - \frac{XY}{N_s + 10 + Y} \tag{10}$$

where: $X = 0.66$ and $Y = 4.71 + \frac{1.72}{i}$.

6.2. Brixius equation

Like the VTI equation, the Brixius equation (Brixius, 1987) uses a numeric referred to as the Brixius number (B_n) using CI instead of G as follows (Brixius, 1987):

$$B_n = \frac{CIbd}{W}\left(\frac{1 + 5\delta/h}{1 + 3\frac{b}{d}}\right) \tag{11}$$

The Brixius equation for gross traction coefficient is (Brixius, 1987):

$$\frac{T}{W} = 0.88\left(1 - e^{-0.1B_n}\right)\left(1 - e^{-7.5i}\right) + 0.04 \tag{12}$$

7. Comparison with existing equations

For comparison with existing equations, the proposed equation and predictions by the VTI and Brixius equation on the same powered DROVE dataset are shown in Fig. 9. Records that could not produce predictions with all three equations were removed leaving 1225 records for use after the removal of the braked data. Each of the plots presented in Fig. 9 contains a 1:1 line where the predicted and measured values match and this is the goal of predictive equations. The proposed equation demonstrates greater clustering

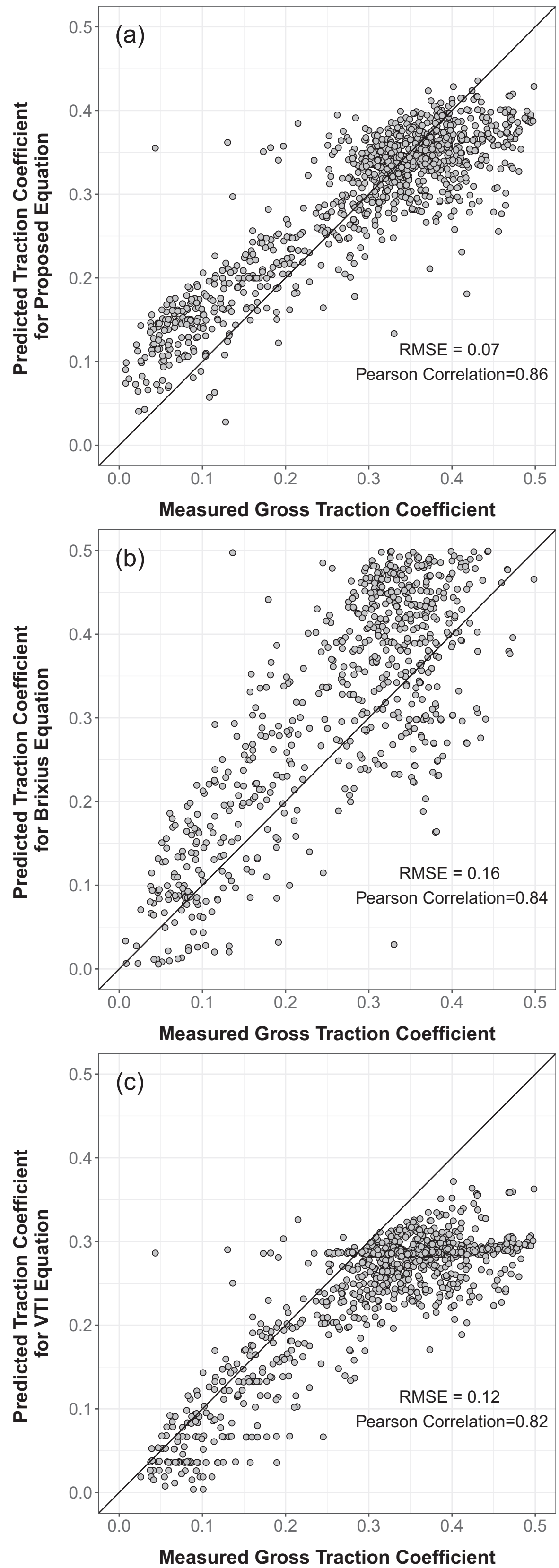

Fig. 9. Predicted versus measured gross traction coefficient for powered tests using: (a) the proposed equation, (b) the Brixius equation, and (c) the VTI equation. Straight line in each graph is the 1:1 (y = x) line.

around the 1:1 line than the Brixius equation or the VTI equation. Due to this improvement in clustering there, the proposed equation presents the lowest RMSE with a value of 0.07 and a relatively high Pearson correlation coefficient of 0.86. The proposed equation tends to over-predict for measured values of 0 to 0.3. For measured values greater than 0.3, the proposed equation appears to have leveled-out with similar numbers of over- and under-prediction. The Brixius equation has a greater amount of error due to the scattering of the data, but still provides acceptable performance with a correlation coefficient of 0.84 and an RMSE of 0.16. There may be a slight tendency to over-predict, but it does not appear to be substantial based on the 1:1 line. The VTI provides similar performance based on the reported RMSE and correlation coefficient (0.12 and 0.82, respectively). However, for measured gross traction coefficients greater than 0.3, the curve begins to veer away from the 1:1 line, trending toward under-predictions. This may be due to application of the VTI outside of the range of values used for its development. Overall, the proposed equation provides a similar performance to the existing equations for powered tests contained in DROVE.

8. Conclusions

A unified traction equation for wheeled vehicles operating in dry sand is proposed in order to allow for traction prediction in all operation modes including towed, braked, and powered. The proposed equation is developed by employing a database of laboratory and field records for wheeled vehicles, entitled Database Records for Off-road Vehicle Environments (DROVE). The equation defines the gross traction coefficient as a function of the slip and with fitting parameters defined as functions of the CP to CI ratio. The primary contribution of this work lies within development of a unified functional form that can estimate traction over all modes of operation. Available data from DROVE are then used to empirically determine fitting relationships for the proposed unified equation.

The proposed model offers an improvement over the existing traction models as it predicts traction over the entire range of braking and powered modes. The proposed continuous traction model compared well with the existing VTI equation and the Brixius equation for powered wheels in regards to RMSE and Pearson correlation while providing an acceptable performance over braked ranges. There is further room for improvement as the proposed equation provides over-prediction for measured gross traction coefficients less than 0.3. This trend for over-prediction is most pronounced for braked operation. The trends shown indicate that the model could be improved by including another related term or correction factor. Overall, the equation performs well and provides a single equation for off-road mobility design and analysis.

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