NUMERICAL CALCULATIONS OF PLANAR SYMMETRIC ARRAYS OF CYLINDRICAL DIPOLES

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Abstract

First part of the work is a theoretical description of the numerical analysis method for finding the current distribution on cylindrical dipoles based on Pocklington's equation solved by the method of momentum, exactly Galerkin's method.

Practical part of the work is a computer program designed for calculations of planar symmetric arrays of cylindrical dipoles based on the above method.

Assumptions of the task

Cylindrical dipoles are placed in direction of the x-axis and are planar symmetric with respect to the yz-plane. They are excited by an incident field, which is constant over the gap and has the same symmetry as the dipoles. Loading network is not interacting with the radiated field.

Another simplification is that we will consider only x-directional currents and that the current is equally distributed along the circumference of each dipole. This simplification is reasonable if the dipole radii are small compared to all other dimensions in the antenna and to wavelength.

Reduction of integral equation to a set of linear equations for finding current distribution

The electrical field integral equation (EFIE) states that the sum of the incident field and the scattered field must satisfy the boundary condition on the dipoles – the tangential component must vanish:

$$\left(\dot{\boldsymbol{E}}^{tot}\right)_t = \left(\dot{\boldsymbol{E}}^{i} + \dot{\boldsymbol{E}}^{s}\right)_t = 0 \qquad \text{on the surface.} \qquad (1)$$

This equation is used for evaluating the current distribution on the dipoles.

The surface current density $\dot{K}^{(k)}(x)$ and the total current $\dot{I}^{(k)}(x)$ are related through

$$\dot{I}^{(k)}(x) = 2\pi a^{(k)}.\ \dot{K}^{(k)}(x) \qquad (2)$$

(where $a^{(k)}$ is the radius of dipole No. k). Currents are x-component and consequently, the vector potential will only have an x-component $\dot{\boldsymbol{A}} = \dot{A}_x = \dot{A}$:

$$\dot{A}(\boldsymbol{r}) = \mu_0 \sum_{k=1}^{n} \iint_{dipole\ k} \dot{K}^{(k)}(x) \frac{exp\left(-jk_0\left|\boldsymbol{r}-\boldsymbol{r}^{(k)}\right|\right)}{4\pi\left|\boldsymbol{r}-\boldsymbol{r}^{(k)}\right|} dS^{(k)} \qquad (3)$$

X-component of the tangential part of $\dot{E}^s$ is

$$\dot{E}_X^S(r) = \frac{1}{j\omega\mu_0\varepsilon_0}\left[\frac{\delta^2 \dot{A}(r)}{\delta x^2} + k_0^2 \dot{A}(r)\right] \tag{4}$$

Thin wire approximation assumes that there is no circumferential variation in fields and potentials:

$$\begin{aligned} \dot{E}_X^{S(k)}(x) &= \dot{E}_X^S\left(r^{(k)}\right) \\ \dot{A}^{(k)}(x) &= \dot{A}\left(r^{(k)}\right) \end{aligned} \tag{5}$$

Using (4) and (5) in (1) we obtain Pocklington's equation for the dipole array:

$$\begin{aligned} &\frac{\delta^2 \dot{A}^{(k)}(x)}{\delta x^2} + k_0^2 \dot{A}^{(k)}(x) = -j\omega\mu_0\varepsilon_0 \dot{E}_x^{i(k)}(x) \\ &k = 1, 2, \dots, n \\ &-h^{(k)} \le x \le h^{(k)} \qquad (h^{(k)} \text{ is half length of dipole No. } k) \end{aligned} \tag{6}$$

Now we take the inner product of each side in (6) with the number of testing functions. On the dipole No. k the following piecewise sinusoidal testing function are used:

$$\begin{aligned} &f_m^{(k)}(x) = \frac{\sin[k_0\left(\Delta x^{(k)} - \left|x - x_m^{(k)}\right|\right)]}{\sin[k_0 \Delta x^{(k)}]} && x_{m-1}^{(k)} \le x \le x_{m+1}^{(k)} \\ &f_m^{(k)}(x) = 0 && \text{otherwise} \quad , \\ &m = 0,\ \pm 1,\ \pm 2, \dots,\ \pm p \end{aligned} \tag{7}$$

where $x_m^{(k)} = m.\Delta x^{(k)}$ is x-position of subdividing point No. m on dipole No. k,
$\Delta x^{(k)} = h^{(k)}/(p+1)$ is the segment length on dipole No. k,
p is number of subdividing points on one half dipole.
The result of the inner product is this difference equation:

$$\begin{aligned} &\dot{A}_{m+1}^{(k)} - 2\cos\left(\delta^{(k)}\right)\dot{A}_m^{(k)} + \dot{A}_{m-1}^{(k)} = -\frac{j\omega\varepsilon_0\mu_0}{k_0}\int_{x_{m-1}^{(k)}}^{x_{m+1}^{(k)}} \dot{E}_x^{i(k)}(x)\sin[k_0\left(\Delta x^{(k)} - \left|x - x_m^{(k)}\right|\right)]dx \\ &k = 1, 2, \dots, n \\ &m = 0, \pm 1, \pm 2, \dots, \pm p \end{aligned} \tag{8}$$

where $\dot{A}_m^{(k)} = \dot{A}^{(k)}\left(x_m^{(k)}\right)$

From (8) a rapidly converging method is obtained using piecewise sinusoidal expansion function for $K^{(k)}(x)$ in (3). This is also known as Galerkin's method. Then

$$\dot{K}^{(k)}(x) = \sum_{m=-p}^{p} \dot{K}_m^{(k)} f_m^{(k)}(x) \qquad \dot{I}^{(k)}(x) = \sum_{m=-p}^{p} \dot{I}_m^{(k)} f_m^{(k)}(x) \tag{9}$$

Using (9) in (3) and defining

$$\dot{S}_{mi}^{kl} = \frac{1}{2\pi a^{(l)}} \iint_{\text{dipole } l} f_i^{(l)}(x) \frac{exp\left(-\mathrm{j}k_0\left|r_m^{(k)} - r^{(l)}\right|\right)}{\left|r_m^{(k)} - r^{(l)}\right|} dS^{(l)} \tag{10}$$

we can obtain from (8), denoting electrical segment length on dipole No. k $\delta^{(k)} = k_0\Delta x^{(k)}$:

$$\sum_{l=1}^{n}\sum_{i=-p}^{p}\left[\dot{S}_{m+1\,i}^{kl} - 2\cos\left(\delta^{(k)}\right)\dot{S}_{mi}^{kl} + \dot{S}_{m-1\,i}^{kl}\right]\dot{I}_i^{(l)} = \dot{U}_m^{(k)} \qquad (11)$$

$$\text{with } \dot{U}_m^{(k)} = -j4\pi\sqrt{\frac{\varepsilon_0}{\mu_0}}\int_{x_{m-1}^{(k)}}^{x_{m+1}^{(k)}}\dot{E}_x^{i(k)}(x)\sin[k_0\left(\Delta x^{(k)} - \left|x - x_m^{(k)}\right|\right)]dx. \qquad (12)$$

From (11) the generalized impedance can be defined as

$$\dot{Z}_{mi}^{kl} \equiv \dot{S}_{m+1i}^{kl} - 2\cos(\delta^{(k)})\dot{S}_{mi}^{kl} + \dot{S}_{m-1i}^{kl} \qquad (13)$$

and we will finish with the generalized impedance equation

$$\sum_{l=1}^{n}\sum_{i=-p}^{p}\dot{Z}_{mi}^{kl}\dot{I}_i^{(l)} = \dot{U}_m^{(k)} \qquad \begin{array}{l} k = 1, 2, \ldots, n \\ m = 0, \pm 1, \pm 2, \ldots, \pm p. \end{array} \qquad (14)$$

Now we will exploit planar symmetry of the antenna: $\begin{array}{l} \dot{I}_{-i}^{(l)} = \dot{I}_i^{(l)} \\ \dot{U}_{-m}^{(k)} = \dot{U}_m^{(k)} \end{array}$ (15)

If we define $\overline{\dot{Z}}_{mi}^{kl} \equiv \dot{Z}_{mi}^{kl} + \dot{Z}_{m-i}^{kl}$ (16) and $\begin{array}{ll} \overline{\dot{I}}_i^{(l)} = \frac{1}{2}\dot{I}_0^{(l)} & \text{for } i = 0 \\ \overline{\dot{I}}_i^{(l)} = \dot{I}_i^{(l)} & \text{for } i \neq 0 \end{array}$, (17)

then because of (17) the system of equation (14) reduces to

$$\sum_{l=1}^{n}\sum_{i=0}^{p}\overline{\dot{Z}}_{mi}^{kl}\overline{\dot{I}}_i^{(l)} = \dot{U}_m^{(k)} \qquad \begin{array}{l} k = 1, 2, \ldots, n \\ m = 0, 1, 2, \ldots, p \end{array} \qquad (18)$$

with $n(p+1)$ unknown currents.

Numerical evaluation of matrix elements

The matrix $\overline{\dot{Z}}$ must be calculated using (16), (13) and (10). The basic problem is numerical evaluation of (10):

$$\dot{S}_{mi}^{kl} = \frac{1}{2\pi a^{(l)}}\int_{x_{i-1}^{(l)}}^{x_{i+1}^{(l)}}\int_0^{2\pi} f_i^{(l)}(x)\frac{\exp\left(-jk_0\left|r_m^{(k)} - r^{(l)}\left(x,\varphi^{(l)}\right)\right|\right)}{\left|r_m^{(k)} - r^{(l)}\left(x,\varphi^{(l)}\right)\right|}a^{(l)}d\varphi^{(l)}dx \qquad (19)$$

If $k = l$, then a two-dimensional integration is used, else one-dimensional, by a maximally 16-points Gaussian integration. Two types of singularities are extracted for $k = l$.

Gap model

The field is assumed to be constant over the gap (half length $g^{(k)}$) and zero outside

$$\dot{E}_x^{i(k)}(x) = \begin{cases} \dfrac{\dot{V}^{(k)}}{2g^{(k)}} & \text{for } -g^{(k)} \le x \le g^{(k)} \\ 0 & \text{otherwise} \end{cases} \qquad (20)$$

This is used in (12) and $U_m^{(k)}$ are evaluated.

Directivity

The far-field formula is used in calculating directivity:

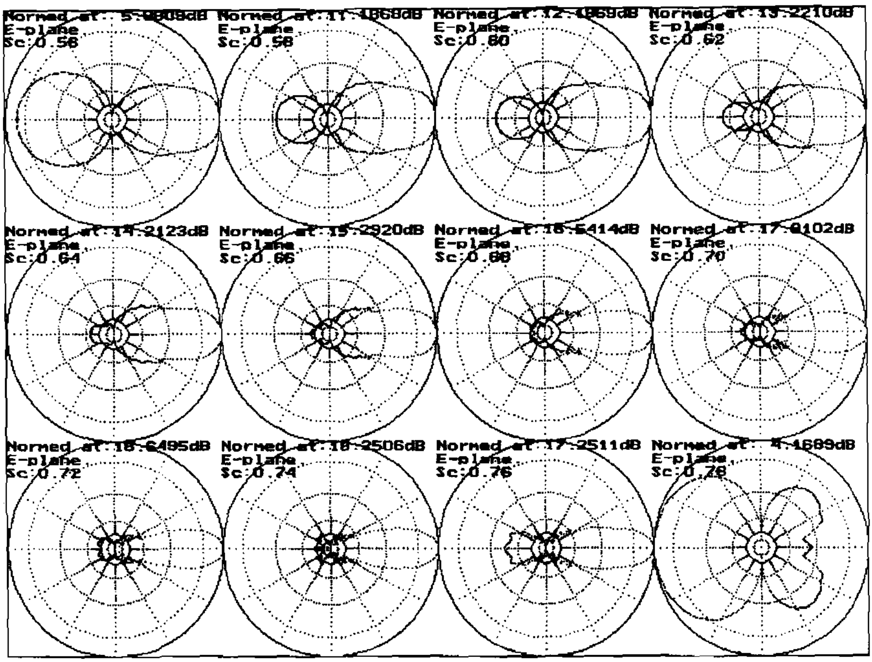
$$\dot{E} = -j\omega[(\dot{A}\cdot\boldsymbol{\Theta}^o)\boldsymbol{\Theta}^o + (\dot{A}\cdot\boldsymbol{\Phi}^o)\boldsymbol{\Phi}^o] \qquad (21)$$

The radiated power per unit angle in the angle direction (ϕ, θ) is evaluated

$$\Phi(\theta,\varphi) = r^2|S(\boldsymbol{r})| = \frac{1}{2}\eta_o r^2|E(\boldsymbol{r})|^2 \quad (S \text{ is Pointing's vector}) \qquad (22)$$

and this value is divided by $P_{rad}/4\pi$ to get the directivity.

Radiating patterns of the 15-dipole array in horizontal plane for different frequencies.



Another approaches

Vector-scalar potential (or two-potential) equation is suitable for solving current distributions of arbitrarily shaped wire structures. But instead of its thin wire approximation, unrealistic gap model, constraints of the expansion functions, inaccurate integration and rounding errors, above described approach is very good for planar symmetric arrays of cylindrical dipoles.

References

[1] Jorgen Hald: Method of moments for arbitrarily loaded planar symmetric arrays of cylindrical diples, report, Electromagnetics institute, DTU 1980